Conclusion

Based on the performed studies, we demonstrate in the monograph that the dispersion refraction effect actually exists. The result generalizing the work consists in that we derived the equations of the modified STRO version for linear media with time dispersion.

In contrast to the standard approach, when a locally plane homogeneous monochromatic wave (which is an exact solution to the wave equation in a homogeneous medium) is used as a field model, we used a more adequate model of a smoothly inhomogeneous wave in the form of the Airy function, which is an exact solution to the wave equation for refraction in a linear layer and is specified by the following equation in our case:

$$\frac{\partial^2 A}{\partial y^2} - 2\left(\frac{\varepsilon_0 \omega}{c^2}\frac{\partial \omega}{\partial y} + \frac{\omega^2}{2c^2}\frac{\partial \varepsilon_0}{\partial y} - \frac{1}{2c^2}\frac{\partial \omega_M}{\partial y}\right) Ay = 0.$$

Thus, we additionally took into account the contribution of the infinite sum of amplitude corrections (4.5), which gave the resultant significant integral correction used to describe refraction, rather than simply considered two terms of the ray series (4.2), as is performed in standard STRO and causes the systematic error.

The obtained approximation explicitly describes the dispersion refraction effect for frequency-modulated waves and corrects the description of ordinary refraction for non-modulated quasi-monochromatic waves in dispersive media.

We now summarize all space-time ray equations obtained for an arbitrary time dispersion law, which is specified by the wave equation (1.1a)–(1.2a):

$$\frac{d\mathbf{r}}{dt} = \mathbf{V}_g;$$

$$\mathbf{V}_g = c^2 \frac{\mathbf{k}}{\varepsilon_0 \omega};$$

$$\frac{d\omega}{dt} = 0;$$

$$\omega_M = \omega^2(\varepsilon_0 - \varepsilon);$$

$$\frac{d\mathbf{V}_g}{dt} = -\frac{c^2}{2\varepsilon_0^2}\nabla\varepsilon_0 + \frac{c^2\omega_M}{2\varepsilon_0^3\omega^2}\nabla_\perp\varepsilon_0 -$$

$$-\frac{c^2}{2\varepsilon_0^2\omega^2}\nabla\omega_M + \frac{c^2\omega_M}{2\varepsilon_0^3\omega^4}\nabla_\perp\omega_M - \frac{c^2\omega_M}{\varepsilon_0^2\omega^3}\nabla_\perp\omega\,;$$

$$\frac{dA}{dt} = -\frac{A_0}{2}\left[\frac{c^2}{\varepsilon_0\omega}D - \left(\frac{1}{k_z} - \frac{c^2k_x}{\varepsilon_0^2\omega^2}\right)\Omega_x + \frac{\omega}{2\varepsilon_0k_x}\frac{\partial\varepsilon_0}{\partial x} - \frac{1}{2\varepsilon_0\omega k_x}\frac{\partial\omega_M}{\partial x}\right];$$

$$\frac{dD}{dt} = -\frac{c^2}{\varepsilon_0\omega}\left(\frac{\omega^2}{2c^2k_x}\frac{\partial\varepsilon_0}{dy} - \frac{1}{2c^2k_x}\frac{\partial\omega_M}{\partial y}\right)^2 - \frac{1}{2\varepsilon_0\omega}\frac{\partial^2\omega_M}{\partial y^2} + \frac{\omega}{2\varepsilon_0}\frac{\partial^2\varepsilon_0}{\partial y^2} -$$

$$-\frac{\omega_M}{\omega(\varepsilon_0\omega^2 - \omega_M)}\Omega_y^2 + \frac{1}{\varepsilon_0\omega^2 - \omega_M}\left(\omega^2\frac{\partial\varepsilon_0}{\partial y} - \frac{\partial\omega_M}{\partial y}\right)\Omega_y - \frac{c^2}{\varepsilon_0\omega}D^2\,;$$

$$\frac{d\Omega_x}{dt} = \frac{1}{k_x}\Omega_y^2 + \left(\frac{1}{\varepsilon_0k_z} - \frac{c^2k_x}{\varepsilon_0\omega^2}\right)\Omega_x^2 -$$

$$-\left(\frac{\omega}{2\varepsilon_0k_x}\frac{\partial\varepsilon_0}{\partial x} - \frac{1}{2\varepsilon_0\omega k_x}\frac{\partial\omega_M}{\partial x}\right)\Omega_x -$$

$$-\left(\frac{\omega}{2\varepsilon_0k_x}\frac{\partial\varepsilon_0}{\partial y} - \frac{1}{2\varepsilon_0\omega k_x}\frac{\partial\omega_M}{\partial y}\right)\Omega_y\,;$$

$$\frac{d\Omega_y}{dt} = \left(\frac{1}{2\varepsilon_0\omega k_x}\frac{\partial\omega_M}{\partial y} - \frac{\omega}{2\varepsilon_0k_x}\frac{\partial\varepsilon_0}{\partial y}\right)\Omega_x - \frac{c^2}{\varepsilon_0\omega}\Omega_yD +$$

$$+\left(\frac{1}{k_x} - \frac{c^2k_x}{\varepsilon_0\omega^2}\right)\Omega_x\Omega_y\,.$$

The following symbols were used here:

$$D = \frac{\partial k_y}{\partial y}\,;\qquad \Omega_x = -\frac{\partial\omega}{\partial x}\,;\qquad \Omega_y = -\frac{\partial\omega}{\partial y}\,,$$

where D is ray divergence, Ω_x is the longitudinal frequency modulation coefficient, and Ω_y is the transverse frequency modulation coefficient.

For the particular case of plasma-like dispersion described by KGE (1.7), these formulas have a simpler form (6.3)–(6.10) presented in Chapter 6.

The total set of ray equations is to a certain degree awkward; however, it becomes possible to artificially control wave in a dispersive medium by forming the wave structure during emission; moreover, these equations make it possible to more accurately describe refraction effects. In addition, these equations are easily numerically integrated, and non-modulated waves can adequately be described by the first five equations.

As was stated above, the standard STRO version can be limitedly used to describe wave refraction in dispersive media. This version can be considered only as a certain asymptotic form of RO for non-modulated waves under the conditions of rather weak dispersion. At the same time, the above equations indicate that the standard STRO version correctly describes refraction in dispersion-free media:

$$\frac{d\mathbf{V}_g}{dt} = -\frac{c^2}{2\varepsilon_0^2}\nabla\varepsilon_0.$$

Below, we will try to demonstrate why a smooth transverse inhomogeneity of the model field is substantial when the RO equations are used to describe waves in a time dispersive media and is inessential for non-dispersive media.

We would like to quote S.M. Rytov: “A paradox exists in wave processes: wave propagation direction depends on phase velocity, whereas energy transfer rate depends on group velocity.”

According to this statement, the ray path curvature radius R in the monochromatic case depends on a relative change in wave number in the transverse direction y:

$$\frac{1}{R} = \frac{\partial k/\partial y}{k}.$$

For a non-dispersive medium, the dispersion relation for which has the form

$$k^2 = \frac{\varepsilon_0\omega^2}{c^2},$$

the refraction value is independent of wave number and frequency:

$$\frac{\partial k/\partial y}{k} = \sqrt{\frac{\partial\varepsilon_0/\partial y}{\varepsilon_0}}.$$

Therefore, any transverse inhomogeneity p of the field amplitude in the form (1.16)

$$U = A_0\exp(-py)\exp\{i(\omega t - kx)\}$$

cannot affect wave refraction because the ray equations remain valid even at $k \to \infty$, when the p/k ratio vanishes.

The situation is absolutely different in a dispersive media. Here wave number cannot be arbitrary and has a quite defined value. The dispersion relation for inhomogeneous waves (1.17)

$$k^2 = \frac{\omega^2}{c^2} - \frac{\omega_L^2}{c^2} + p^2$$

shows that transverse inhomogeneity *p,* together with wave frequency ω and medium eigenfrequency ω_L, enters into the expression for *k*. Refraction can be observed in a medium only when ω and ω_L values are comeasurable. When $\omega_L \to \omega$, wave number $k \to 0$, and the p/k ratio can generally be arbitrarily large. Within the scope of RO, the $p/\kappa \sim 1/L_W \sim O(\chi)$ value substantially changes wave phase velocity and corresponds to refractive phenomena, which is taken into account in the proposed STRO version. This becomes obvious when the exact solutions to KGE (1.18)–(1.20) are carefully analyzed.

At a further increase in p/κ, we fall outside the scope of RO (4.1) into the field of complex RO, which can take diffractive wave effects into account.

The results obtained in this monograph can be interpreted in two ways: (1) we widened the RO possibilities, or (2) these possibilities are now simply within the natural framework. The second version is probably more correct because the STRO applicability conditions (4.1) have long been declared and generally recognized, and other conditions, e.g., in the form of limitation of the infinite ray series by two main terms, have not been introduced according to the author's opinion.

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