

SOME FRACTAL PROPERTIES OF SOLAR MAGNETIC FIELDS

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ABSTRACT

In this paper, we discuss some results of the study of spatial characteristics of solar magnetic fields. The analysis is based on the magnetic field data obtained with a new spectromagnetograph installed on the IZMIRAN Tower Telescope (FeI 6103 Å) (Kozhevnikov et al., 2002), the data of the MSFC solar vector magnetograph (FeI 5250.2 Å) and the data of longitudinal magnetic 96 m daily maps of SOHO/MDI magnetograph (NiI 6768 Å) downloaded through Internet. Our study was concentrated on two objects: the fractal properties of sunspots and fractal properties of the space distribution of the magnetic fields along great distances comparable with the size of great active regions or active complexes. To investigate the fractal structure of the sunspots we used the well known method of calculating a fractal dimension using the relation between the area and the perimeter of magnetic field lines (see (Feder, 1988; Meunier, 1999; Nesme-Ribes et al., 1996; Balke et al., 1993)) and for analysis of fractal properties of the space magnetic field distribution on the solar surface the technique developed by Higuchi, 1988 (who applied it to the investigation of long time series) was used. Note also that sunspot magnetic structure in terms of the fractal models was developed earlier in (Zelenyi, Milovanov, 1991; Milovanov, Zelenyi, 1993; Mogilevskii, 1994; Mogilevskii, 2001), where the sunspots were represented as clusters of the magnetic flux tubes described by the fractal geometry.

1. THE FRACTAL PROPERTIES OF SUNSPOTS

According to (Mandelbrot, 1983), fractal dimensions of the contour lines can be defined from the relation between its perimeter L and area S

$$L \sim S^{D/2} \quad (1)$$

or

$$2 \log L = D \log S + a \quad (2)$$

where parameter D (called Hausdorff dimension) is characteristic of the contour complexity (its denticulation). For instant, for the circle D is equal to 1

whereas for the very jagged contour it is close to 2. If the relation between $2 \log L$ and $\log S$ for some set of contours is linear we can define them as self-similarity objects.

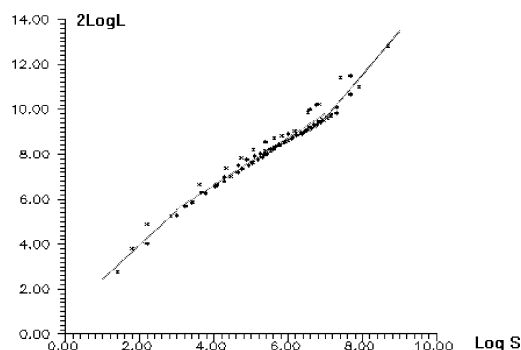


Figure 1. Relation between perimeters L and enclosed areas for longitudinal magnetic field intensity contours in the interval 2200-400 G (summary diagram for 20 sunspots).

In Fig. 1, the summary diagram ($2 \log L / \log S$) based on the estimates of the corresponding properties of the longitudinal magnetic field intensity contour lines for about 20 spot plotted at 200 G steps in the range of 2200-400 G is shown. One can see that the relationship under discussion is well approximated by three straight lines, first one corresponding to the $\log S$ range 1–3 ($D=1.52$), the second one to the range 3-7.2 ($D=1.05$) and the third one to the range 7.2 – 9 ($D=1.93$). The similar distribution (with somewhat different intervals) was obtained also for the contour intensity lines of total magnetic field (Fig. 2).

The comparison with the photospheric brightness distribution on the respective maps shows that the first line approximately corresponds to the sunspot umbra, the second line – to the sunspot penumbra, and the third one to the ambient photosphere. So we can suggest the existence of three families of self-similar contour lines roughly corresponding to the umbra, penumbra and the ambient photosphere correspondingly. Note that the greatest fractal dimension corresponds to the region of the weakest fields (ambient photosphere), the least one corresponds to the intermediate region (penumbra).

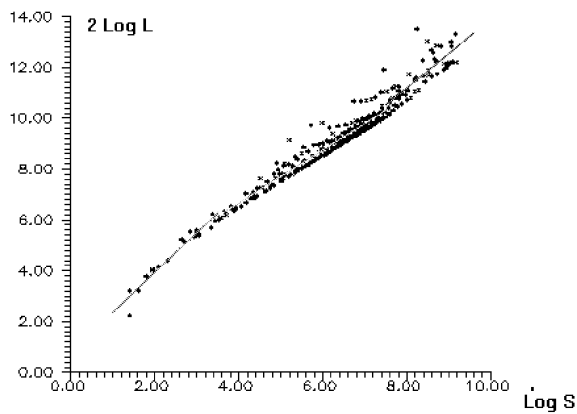


Figure 2. Relation between the perimeters L and enclosed areas for full magnetic field intensity contours l in the interval 2200-400 G (summary diagram for 20 sunspots).

To study the peculiarities of the sunspot magnetic structure in terms of the fractal models (see (Zelenyi, Milovanov, 1991; Milovanov, Zelenyi, 1993; Mogilevskii, 1994; Mogilevskii, 2001)) in more detail we used the MSFC magnetic data of some active groups near the center of the solar disk (27.11.2000, N08 E22; 16.07.2002, N18 E00; 19.11.2003, N03 E09; 23.10.2003, N10 W06 (two spots); 24.07.2004, N08 W10; 18.08.2002, S07 W03), selected the largest spot in each group and restricted our analysis to the area with the same field polarity as the spot.

For all spots we have determined the length of the field lines and the areas they enclose in the range of the magnetic

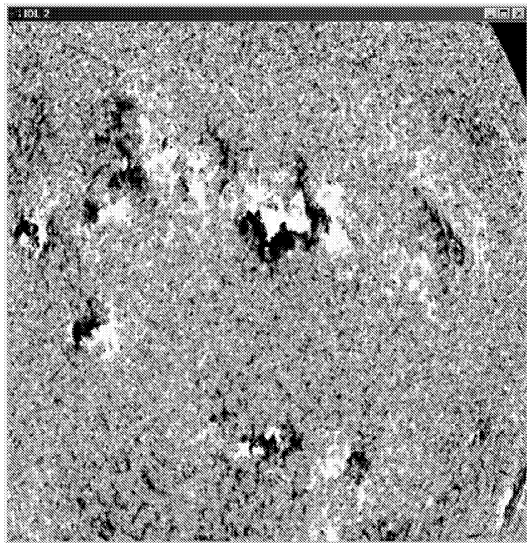


Figure 4. The longitudinal magnetic field of June 14, 2000 (SOHO/MDI), used to obtain the fractal index of space distribution of magnetic fields along scans corresponding to different mean magnetic intensities

field intensities from 2200 to 400 G (at 200 G steps). Then, we plotted $2\log L$ versus $\log S$ for all field lines of a given magnetic intensity, no matter which sunspot or sunspot group they belonged to. All points for each intensity value B were well approximated by linear dependence with the angular coefficient d .

Fig. 3 shows a plot of the fractal dimension d versus magnetic field intensity based on calculations for all sunspots under consideration and standard errors. One can see that the fractal coefficient has a maximum (close to 1.50) approximately corresponded to the longitudinal intensity 1200 G (close to the umbra-penumbra interface). The value $d=1.50$ is close to the value obtained by Zelenyi and Milovanov (1991) by a different method. Note also the increasing of fractal numbers towards the ambient photosphere.

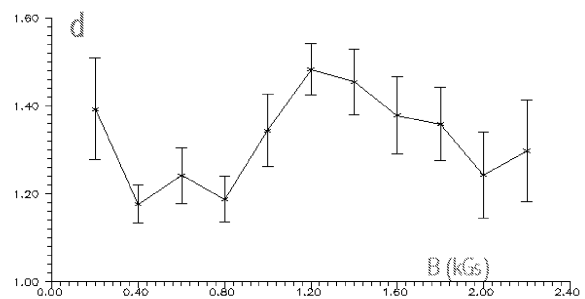


Figure 3. Fractal dimension as a function of the intensity of the longitudinal magnetic field.

It is possible that these peculiarities may signify the existence of turbulent currents at the umbra - penumbra and at the penumbra - photosphere interfaces.

2. THE FRACTAL PROPERTIES OF THE SPACE MAGNETIC FIELD DISTRIBUTION ON THE SOLAR SURFACE

As we said above for analysis of fractal properties of the space magnetic field distribution on the solar surface we used the technique for calculating the fractal dimensions developed by Higuchi (1988), who applied it to the investigation of long time series.

$$L_m(k) = \left\{ \left[\sum_{i=1}^{\left[\frac{N-m}{k} \right]} X(m+ik) - X(m+(i-1)\cdot k) \right] \frac{N-1}{\left[\frac{N-m}{k} \right] \cdot k} \right\} / k. \quad (3)$$

Here X is the measured parameter (for example magnetic field, velocity or intensity), N – the number of point in the sequence, m and k indicate the initial point and the interval. Than the log of the average value of “length of the curve” L_m is compared with $\log(k)$. The coefficient of proportionality D in the dependence $\log\langle L(k) \rangle = -D \cdot \log(k) + C$ is assumed here as a

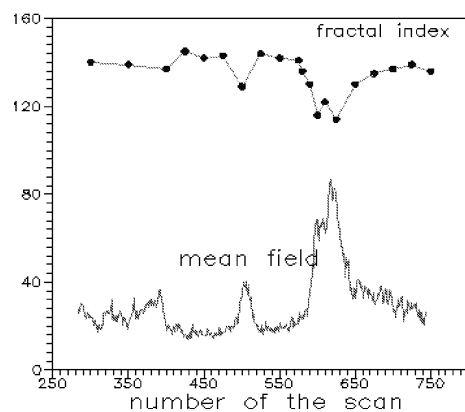


Fig. 5. The relation between the spatial fractal index and mean magnetic field corresponding to different scans of the map presented on Fig. 4

fractality index. We investigate the spatial distribution of magnetic fields along various scans of longitudinal magnetic field 96-m maps SOHO/MDI passing through both the quiet and active solar regions. As an example we present the results obtained from the analysis of the longitudinal field map obtained with SOHO MDI for June 14 2000 (Fig. 4).

Fig. 5 illustrates the fractal index (multiplied by 100) calculated in every 25-th scan along with the mean absolute value of the longitudinal field in every scan. The good correlation of these two curves is clearly seen. This result is likely to suggest that, in quiet region, the field is very complicated and nearly randomly distributed. In active regions, the field is much more regular.

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REFERENCES

- Balke A.C., et al., *Solar Physics.*, Vol. 143, 215-227, 1993.
- Feder J., *Fractals*, Plenum, New York. 1988
- Higuchi T., *Physica*, D 31, 277-283, 1988
- Kozhevator I.E. et al., *Instruments and Experimental Techniques*, Vol. 45, No 1, 98—102, 2002
- Mandelbrot B.B., *The Fractal Geometry of Nature*, New York, Freeman, 1983
- Meunier N., *Astrophys. J.*, Vol. 515, 801—811, 1999
- Milovanov A.V., and Zelenyi L.M., *Phys. Fluids*, Vol. B 5(7), 2609-2615, 1993
- Mogilevskii E.I., *Soviet Astron. Lett.*, Vol. 20, N8, 607—612, 1994.

- Mogilevskii E.I., *Fractals on the Sun*, Fizmatlit., 2001.
- Nesme--Ribes E., et al. *Astron. Astrophys.*, 308, 213—218, 1996.
- Zelenyi L.M. and Milovanov A.V., *Soviet Astronomical Letters*, Vol. 17, 425, 1991.